Study of heat and mass transport in a temperature dependent viscosity fluid layer under temperature modulation

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Abstract—In this paper, we study the thermosolutal convection in a horizontal temperature dependant viscous fluid layer. The considered temperature profile consists of two parts: a steady part and a time-dependent periodic part that oscillates with time. A weak nonlinear stability analysis has been performed by using power series expansion in terms of the amplitude of temperature modulation, which is assumed to be small. The Nusselt and Sherwood numbers have been obtained in terms of the amplitude of convection which is governed by the non autonomous Ginzburg-Landau equation derived for the stationary mode of convection. Effects of various parameters such as frequency and amplitude of modulation, Prandtl number, diffusivity ratio and solute Rayleigh number, have been analyzed on heat and mass transfer. It is found that heat and mass transport can be controlled by suitably adjusting the external parameters of the system. It is also found that the thermo-rheological parameter is to destabilize the system.

Index Terms—Temperature modulation, Heat and mass transfer, Ginzburg-Landau equation, Temperature-dependant viscosity.

1 INTRODUCTION

Double diffusive convection is an important fluid dynamics phenomenon. It is a type of instability that occurs in a fluid that possesses two opposing density altering components with differing molecular diffusivity, such as heat and salt or any two solute concentrations. The marked difference between single and double diffusive systems lies in the verity that in double diffusive systems convection can occur even when the system is hydrostatically stable if the diffusivities of the two diffusing fields are widely different. The study of the double diffusive convection has received much attention over the years due to its numerous fundamental and industrial applications. Some examples of double diffusive convection can be found in oceanography Stommel (1956), lakes and underground water, atmospheric pollution, chemical processes, laboratory experiments, modeling of solar ponds, astrophysics, geophysics, geology and engineering Chen and Johnson (1984), magma chambers and sparks, formation of microstructure during the cooling of molten metals, fluid flows around shrouded heat-dissipation fins, migration of moisture through air contained in fibrous insulations, grain storage system, the dispersion of contaminants through water saturated soil, crystal growth, solidification of binary mixtures, and the underground disposal of nuclear wastes. The early work on this problem is summarized in several reviews; Turner (1973-1974), Huppert and Turner (1981), Bhadauria (2006).

Stommel et al. (1956) were the first to notice some properties of double diffusive convection with the discovery of the phenomenon of the salt fountain, which occurs when hot salty water lies above cold fresh water. Such a system was later analyzed by Stern (1960), who noted the general properties of the motion, now commonly known as salt fingers. The situation with reversed gradients has been studied by Veronis (1965), and stability criteria for horizontal boundaries of various kinds have been presented by Nield (1967) by means of a linear stability analysis. Lortz (1965) studied the effect of magnetic field on double diffusive convection. His object was to clarify some of the mathematical aspects of stability criterion (Malkus and Veronis (1958)) but, his analysis is silent about the detailed study of stability analysis. Considering linear gradients, Baines and Gill (1969) investigated the thermohaline convection in a fluid layer confined between two horizontal boundaries, which are dynamically free and conducting to both heat and salt. Chen (1974) considered a two-dimensional problem of a linearly stratified salt solution contained between two infinite vertical plates, and studied the onset of cellular convection due to a lateral temperature gradient. Proctor (1981) studied the thermohaline convection in a horizontal fluid layer using rigid-rigid and free-free boundaries. Double diffusive convection in an inclined plane was investigated by Thangam et al. (1982) for rigid-rigid boundaries. Later on many other investigators studied this problem of double diffusive convection under various physical and boundary conditions.

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Sodha and Kumar (1985) studied the stability of double diffusive convection in solar ponds with non-constant temperature and salinity gradients. Lopez et al. (1990) have performed a linear stability analysis of triple diffusive convection in a horizontal fluid layer and found the effect of rigid-rigid boundaries on the onset of convection. Using linear stability analysis, Saunders et al. (1992) studied the effect of gravity modulation on thermosolutal convection in an infinite layer of fluid using free-free boundaries. Gobin and Bennacer (1994) investigated the problem of thermohaline convection in a vertical layer of a binary fluid, and studied the onset of convection. Sezai and Mohamad (2000) have performed a three-dimensional numerical study to investigate double diffusive, natural convection in a cubic enclosure subject to opposing and horizontal gradients of heat and solute imposed along the two vertical side walls. Ryskin et al. (2003) have studied thermo-diffusive convection in ferrofluids. Bhadauria (2006) analyzed this problem by considering rigid boundaries under temperature modulation. Starchenko (2006) discussed double diffusion magneto-convection for Earths type planets. Siddheshwar et al. (2012) performed a local non-linear stability analysis of Rayleigh-Be'nard magneto-convection using Ginzburg-Landau equation. They showed that gravity modulation can be used to enhance or diminish the heat transport in stationary magneto-convection. Magnetohydrodynamic natural convection boundary layer flow of an electrically conducting and viscous incompressible fluid along a heated vertical flat plate with uniform heat and mass flux in the presence of strong cross magnetic field has been investigated by Sadia (2012).

The classical Rayleigh-B'enard convection due to bottom heating is well known and highly explored phenomenon given by Chandrasekhar (1961) and Drazin and Reid (2004). The basic state temperature gradient across a fluid layer has to be time dependent and space dependent, and this can be used to regulate convection through an external means. Venezian (1969) was the first to study the effect of temperature modulation on thermal instability in a horizontal fluid layer, as a thermal analogue of Donnely (1964) experiments. Using perturbation method and considering free-free surfaces, he calculated the shift in the critical Rayleigh number and showed that the system can be stabilized or destabilized by suitably tuning the frequency of modulation. A similar problem was studied by Gershuni and Zhukhovitskii (1963) for a temperature profile obeying rectangular law. Rosenblat and Herbert (1970)} investigated thermal instability for low frequency temperature modulation while Rosenblat and Tanaka (1971), Yih, Li CH, (1972) and Kumar et al.(1986) studied the effect of thermal modulation on the onset of Rayleigh-B'enard convection with rigid boundaries, using Galerkin technique and discussed the stability of the system using Floquet theory. The Venezian problem for free-free surfaces was extended by Finucane and Kelly (1976), Roppo et al. (1984), for weakly nonlinear thermal instability under temperature modulation. They observed that stable hexagons are produced by the modulation effect near the critical Rayleigh number. Considering various temperature profiles, Bhadauria (2006), Bhadauria and Bhatia (2002), studied temperature modulation of Rayleigh-B'enard convection for rigid-rigid boundaries. Malashetty and Swamy (2008) investigated thermal instability of a heated fluid layer subject to both boundary temperature modulation and rotation. They found that the symmetric modulation destabilizes the system at low frequencies but stabilizes at moderate and high frequencies. Asymmetric modulation was shown to stabilize convection for all frequencies. A weakly nonlinear study on thermal instability with temperature modulation using Lorenz model was made by Bhadauria et al. (2009) considering various temperature profiles. In addition to finding the effect of temperature modulation, they compared the results of various temperature profiles, in terms of the critical Rayleigh number. Raju and Bhattacharyya (2010) investigated onset of thermal instability in a horizontal layer of fluid with modulated boundary temperatures by considering rigid boundaries. Siddheshwar et al.(2012) stationary magneto convection in a Newtonian liquid under temperature or gravity modulation using Ginzburg-Landau model. Bhadauria et al. (2012), investigated a non-linear thermal instability in a rotating viscous fluid layer under temperature/gravity modulation.

Most of the above studies considered only constant viscosity; however, in nature and in engineering problems of convective flow, viscosity of many fluids varies with temperature. Therefore, the results drawn from the flow of fluids with constant viscosity are not applicable for the fluid that flows with temperature dependent viscosity, particularly at high temperature. The fluids that flow with variable viscosity are useful in chemical, bio-chemical and process industries as well as in physics of fluid flows, wherein the flow of fluids is governed by different temperatures. Therefore, the objective of our study is to investigate the effect of temperature modulation on double diffusive convection in a horizontal fluid layer. Here, we have investigated heat and mass transfer using

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nonlinear stability analysis, and the results were presented graphically in terms of Nusselt, Nu and Sherwood number, Sh respectively.

2. Governing Equations

We consider an infinite horizontal layer of temperature dependent viscous fluid mixture subjected to a vertical gravity field, confined between two free-free boundaries at z=0 and z=d. To maintain a constant temperature difference ΔT and a constant solutal difference ΔS, across the layer, the layer is heated and salted from below. A Cartesian frame of reference is chosen with origin in the lower boundary and the z-axis vertically upwards. The schematic diagram of the problem (temperature modulation) is shown in the Fig.1 given below. The fluid layer is considered to be Boussinesq, and thus the basic governing equations are

$$\nabla \cdot q = 0, \tag{1}$$

$$\rho_0\left(\frac{\partial q}{\partial t} + (q\cdot\nabla)q\right) = -\nabla p + \rho g\hat{k} + \mu(T)\nabla^2 q, \tag{2}$$

$$\gamma\frac{\partial T}{\partial t} + (q\cdot\nabla)\mathrm{T} = \kappa_T \nabla^2 T, \tag{3}$$

$$\frac{\partial S}{\partial t} + (q\cdot\nabla)\mathrm{S} = \kappa_S \nabla^2 S, \tag{4}$$

$$\rho = \rho_0[1-\beta_T(T-T_0)+\beta_S(\mathrm{S}-S_0)], \tag{5}$$

$$\mu(T) = \frac{\mu_0}{1+\varepsilon^2\delta_0(T-T_0)}. \tag{6}$$

The constants and variables used in the above Eqs. (1-6) have their usual meanings, and are given in the Nomenclature. The thermo-rheological relationship in Eq.[6] is guided by Nield (1996).

3. Mathematical Formulation

Since an uniform concentration gradient $\frac{\Delta S}{d}$, has been maintained between the walls of the liquid layer, therefore the boundary conditions on S are

$$\begin{aligned} S &= S_0 + \Delta S \quad &\text{at z=0} \\ &= S_0 \quad &\text{at z=d} \end{aligned} \tag{7}$$

Also the externally imposed surface temperature conditions are

$$\begin{aligned} \mathrm{T} &= \mathrm{T}_0 + \Delta T\left[1+\varepsilon^2\delta\cos(\omega t)\right] \quad &\text{at z=0} \\ &= \mathrm{T}_0 + \Delta T\varepsilon^2\delta\cos(\omega t+\theta) \quad &\text{at z=d} \end{aligned} \tag{8}$$

Here δ represents the amplitude of modulation and ε (to be defined later in the next section) indicates the smallness of the amplitude, ΔT is the temperature difference, ω is the modulation frequency and θ is the phase angle. The basic state of liquid is quiescent and given by

$$q = q_b(\mathrm{z}),\ \rho=\rho_b(\mathrm{z,t}),\ \mathrm{p}=\mathrm{p}_b(z,t),\ ,\ \mathrm{S}=\mathrm{S}_b(z),\ \mathrm{T}=\mathrm{T}_b(z,t) \tag{9}$$

$$\frac{d^2 S_b}{dz^2} = 0, \tag{10}$$

$$\frac{\partial T_b}{\partial t} = \kappa_T\frac{\partial^2 T_b}{\partial z^2}, \tag{11}$$

$$\frac{\partial p_b}{\partial z} = -g\rho_b \tag{12}$$

$$\rho_b = \rho_0[1-\beta_T(T_b-T_0)+\beta_S(\mathrm{S}_b-S_0)], \tag{13}$$

The solution of the Eq.(10) subject to the boundary conditions Eq.(7) is given by

$$S_b = S_0 + \Delta S\left(1-\frac{z}{d}\right), \tag{14}$$

Also the Eq.(11) has been solved subjected to the thermal boundary conditions Eq.(8), we write

where $T_s(\mathrm{z}) = T_0 + \Delta T\left(1-\frac{z}{d}\right)$, (15)

$$\mathrm{T}_b(\mathrm{z,t}) = \mathrm{T}_s(z) + \varepsilon^2\delta_1\,\mathrm{Re}\{\mathrm{T}_3(\mathrm{z,t})\}, \tag{16}$$

$$\mathrm{T}_3(\mathrm{z,t}) = [\mathrm{a}(\lambda)e^{\frac{\lambda z}{d}} + \mathrm{a}(-\lambda)e^{-\frac{\lambda z}{d}}]e^{-i\omega t}, \tag{17}$$

$$\mathrm{a}(\lambda) = \Delta T\frac{e^{-i\theta}-e^{-\lambda}}{e^{-\lambda}-e^{-\lambda}} \text{ and } \lambda^2 = \frac{-i\omega d^2}{\kappa_T}. \tag{18}$$

In the above equations, $\mathrm{T}_s(z)$ is the steady temperature field and T_3 is the oscillating part of T_b, while **Re** stands for the real part. We assume finite amplitude perturbations to the basic state in the form

$$q = q_b + q',\ \rho=\rho_b+\rho',\ \mathrm{p}=\mathrm{p}_b+\mathrm{p}',\ \mathrm{S}=\mathrm{S}_b+\mathrm{S}',\mathrm{T}=\mathrm{T}_b+\mathrm{T}' \tag{19}$$

Substituting Eq.(19) in the set of Eqs.(1-6), we get the following equations

$$\nabla \cdot q' = 0, \tag{20}$$

$$\rho_0\left(\frac{\partial q'}{\partial t} + (q'.\nabla)q'\right) + \nabla\mathrm{p}' = \rho'\mathrm{g}\hat{k} + \mu(T)\nabla^2 q', \tag{21}$$

$$\gamma\frac{\partial \mathrm{T}'}{\partial t} + (q'\cdot\nabla)\mathrm{T}' + \mathrm{w}'\frac{\partial T_b}{\partial z} = \kappa_T\nabla^2\mathrm{T}', \tag{22}$$

$$\phi\frac{\partial \mathrm{S}'}{\partial t} + (q'\cdot\nabla)\mathrm{S}' + \mathrm{w}'\frac{\partial S_b}{\partial z} = \kappa_S\nabla^2\mathrm{S}', \tag{23}$$

$$\rho_0 = -(\beta_T\mathrm{T}' - \beta_S S'). \tag{24}$$

We consider only two-dimensional disturbances in our study, and hence the stream function ψ is introduced as $u' = \frac{\partial \psi}{\partial z}, w' = -\frac{\partial \psi}{\partial x}$. (25)

By operating curl twice on Eq.(20), we eliminate p' from it, and use Eq.(23) to eliminate ρ', and then render the resulting equation and Eqs.(20-24), and (22) dimensionless using the following transformations $(x', y', z') = d(x^*, y^*, z^*)$, $t' = \frac{d^2}{\kappa_T} t^*$

$$q' = \frac{\kappa_T}{d} q^*, \; S' = \Delta S \; S^*, \psi = \kappa_T \psi^* \; T' = \Delta T \; T^*$$

and $\omega^* = \frac{\kappa_T}{d^2}\omega$. We obtain the non-dimensional governing equations in the form (on dropping the asterisks for simplicity)

$$\frac{1}{\Pr}\frac{\partial \nabla^2 \psi}{\partial t} - \frac{1}{\Pr}\frac{\partial(\psi, \nabla^2\psi)}{\partial(x,z)} + Ra_T \frac{\partial T}{\partial x} = \bar{\mu}(T)\nabla^4 \psi + \frac{\partial \bar{\mu}}{\partial z}\frac{\partial \nabla^2 \psi}{\partial z} + Ra_S \frac{\partial S}{\partial x}, \tag{26}$$

$$-\frac{\partial \psi}{\partial x}\frac{\partial T_b}{\partial z} - \nabla^2 T = -\frac{\partial T}{\partial t} + \frac{\partial(\psi, T)}{\partial(x,z)} \tag{27}$$

$$\frac{\partial \psi}{\partial x} - \frac{1}{Le}\nabla^2 S = -\frac{\partial S}{\partial t} + \frac{\partial(\psi, S)}{\partial(x,z)} \tag{28}$$

where $\bar{\mu}(T) = \frac{1}{1+\varepsilon^2 VT}$, ε^2 is a small quantity which indicates that the viscosity variation with temperature is weak, V is the temperature dependent viscosity. The non-dimensionalized parameters in the above equations are: $\Pr = \frac{\nu}{\kappa_T}$ is the Prandtl number,

$Ra_T = \frac{\beta_T g \Delta T K d^3}{\nu \kappa_T}$ is the thermal Rayleigh number,

$Ra_S = \frac{\beta_S g \Delta T K d^3}{\nu \kappa_T}$ is the solute Rayleigh number,

and $Le = \frac{\kappa_T}{\kappa_S}$ is the Lewis number.

The non-dimensional form of temperature gradient in Eq.(27) shows that the basic state solution influences the stability problem through the factor $\frac{\partial T_b}{\partial z}$, which is given by

$$\frac{\partial T_b}{\partial z} = -1 + \varepsilon^2 \delta [f_2(z,t)], \tag{29}$$

where $f_2 = \mathrm{Re}[f e^{-i\omega t}]$, (30)

$$f = [a(\lambda)e^{\lambda z} + a(-\lambda)e^{-\lambda z}],$$

$a(\lambda) = \lambda \frac{e^{-i\theta} - e^{-\lambda}}{e^{-\lambda} - e^{-\lambda}}$ and $\lambda^2 = (1-i)\sqrt{\frac{\omega}{2}}$.

To keep the time variation slow, we have re-scaled the time 't' by using the time scale $\tau = \varepsilon^2 t$. Now, to study the stationary mode of double-diffusive convection, we write the above non-linear system Eqs.(25)-(26) into the matrix form

$$\begin{bmatrix} -\bar{\mu}\nabla^4 & Ra_T \frac{\partial}{\partial x} & -Ras\frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & -\frac{1}{Le}\nabla^2 \end{bmatrix} \begin{bmatrix} \psi \\ T \\ S \end{bmatrix} = \begin{bmatrix} -\frac{\varepsilon^2}{\Pr}\frac{\partial \nabla^2 \psi}{\partial t} + \frac{1}{\Pr}\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,z)} + \frac{\partial \bar{\mu}}{\partial z}\frac{\partial \nabla^2 \psi}{\partial z} \\ \frac{\partial(\psi, T)}{\partial(x,z)} - \varepsilon^2 \frac{\partial T}{\partial \tau} + \varepsilon^2 \delta f_2 \frac{\partial \psi}{\partial x} \\ \frac{\partial(\psi, S)}{\partial(x,z)} - \varepsilon^2 \frac{\partial S}{\partial \tau} \end{bmatrix} \tag{31}$$

We solve Eq.(31) by using $\bar{\mu} = \bar{\mu}(T_b)$ guided by Nield (1996) and considering free-free, isothermal and isohaline boundary conditions as given bellow

$\psi = \nabla^2 \psi = 0, \quad T = S = 0$ on z=1 (32)

$\psi = \nabla^2 \psi = 0, \quad T = S = 1$ on z=0. (33)

3 Heat and mass transport for stationary instability

To solve the system Eq.(31) we introduce the following asymptotic expansion

$$Ra_T = Ra_{0c} + \varepsilon^2 R_2 + \ldots \tag{34}$$

$$\psi = \varepsilon \psi_1 + \varepsilon^2 \psi_2 + \varepsilon^3 \psi_2 + \ldots \tag{35}$$

$$T = \varepsilon T_1 + \varepsilon^2 T_2 + \varepsilon^3 T_3 + .. \tag{36}$$

$$S = \varepsilon S_1 + \varepsilon^2 S_2 + \varepsilon^3 S_3 + .. \tag{37}$$

where Ra_{0c} is the critical value of the Rayleigh number at which the onset of convection takes place in the absence of temperature modulation.

At the lowest order, we have

$$\begin{bmatrix} -\nabla^4 & Ra_{0c}\frac{\partial}{\partial x} & -Ras\frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & -\frac{1}{Le}\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ T_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} s \tag{38}$$

The solution of the lowest order system subject to the boundary conditions Eq.(32-33), is

$$\psi_1 = A\sin(k_c x)\sin(\pi z),$$

$$T_1 = -\frac{k_c}{\alpha^2} A\cos(k_c x)\sin(\pi z),$$

$$S_1 = -\frac{k_c Le}{\alpha^2} A\cos(k_c x)\sin(\pi z), \quad (39)$$

where $\alpha^2 = k_c^2 + \pi^2$. The system (38) gives us the critical value of the Rayleigh number and the corresponding wave number for the onset of stationary convection

$$Ra_{0c} = \frac{\alpha^6 + Lek_c^2 Ras}{k_c^2}, \quad (40)$$

$$k_c = \frac{\pi}{\sqrt{2}}, \quad (41)$$

which is the classical results obtained by Chandrasekhar (1961).

At the second order, we have

$$\begin{bmatrix} -\nabla^4 & Ra_{0c}\frac{\partial}{\partial x} & -Ras\frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & -\frac{1}{Le}\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_2 \\ T_2 \\ S_2 \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \\ R_{23} \end{bmatrix} \quad (42)$$

where

$$R_{21} = 0 \quad (43)$$

$$R_{22} = \frac{\partial \psi_1}{\partial x}\frac{\partial T_1}{\partial z} - \frac{\partial \psi_1}{\partial z}\frac{\partial T_1}{\partial x} \quad (44)$$

$$R_{23} = \frac{\partial \psi_1}{\partial x}\frac{\partial S_1}{\partial z} - \frac{\partial \psi_1}{\partial z}\frac{\partial S_1}{\partial x} \quad (45)$$

The second order solutions subjected to the boundary conditions Eq.(32-33), is obtained as where

$$\psi_2 = 0 \quad (46)$$

$$T_2 = -\frac{k_c^2 A^2}{8\pi\alpha^2}\sin(2\pi z), \quad (47)$$

$$S_2 = -\frac{k_c^2 Le^2 A^2}{8\pi\alpha^2}\sin(2\pi z). \quad (48)$$

The horizontally-averaged Nusselt number, Nu, and Sherwood number, Sh, for the stationary double-diffusive convection (the mode considered in this problem) are given by

$$Nu(\tau) = 1 + \frac{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}} \frac{\partial T_2}{\partial z}dx\right]_{z=0}}{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}} \frac{\partial T_b}{\partial z}dx\right]_{z=0}}$$

$$Sh(\tau) = 1 + \frac{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}} (1 - z + S_2)dx\right]_{z=0}}{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}} (1 - z)dx\right]_{z=0}}$$

$$\text{or } Nu(\tau) = 1 + \frac{k_c^2 A^2}{4\alpha^2} \quad (49)$$

$$\text{and } \quad Sh(\tau) = 1 + \frac{Le^2 k_c^2 A^2}{4\alpha^2} \quad (50)$$

At the third order, we have

$$\begin{bmatrix} -\nabla^4 & Ra_{0c}\frac{\partial}{\partial x} & -Ras\frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & -\frac{1}{Le}\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_3 \\ T_3 \\ S_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \end{bmatrix} \quad (51)$$

where

$$R_{31} = -\frac{1}{\Pr}\frac{\partial}{\partial \tau}\nabla^2\psi_1 - VT_b\nabla^2\psi_1 - (2Ra_{0c}V\,T_b + R_2)\frac{\partial T_1}{\partial x}$$
$$+2\,Ras\,VT_b\frac{\partial S_1}{\partial x} + V\frac{\partial}{\partial z}\nabla^2\psi_1$$

$$R_{32} = -\frac{\partial T_1}{\partial \tau} + \frac{\partial T_2}{\partial z}\frac{\partial \psi_1}{\partial x} + \delta f_2 \frac{\partial \psi_1}{\partial x},$$

$$R_{33} = -\frac{\partial S_1}{\partial \tau} + \frac{\partial S_2}{\partial z}\frac{\partial \psi_1}{\partial x}.$$

Using the first and second order solutions we can easily obtain the expression for R_{31}, R_{32} and R_{33}. Now applying solvability condition for the existence of third order solution, we obtain the Ginzburg-Landau equation for stationary convection with time periodic coefficients in the form

$$Q_1\frac{\partial A(\tau)}{\partial \tau} = Q_2\,A(\tau) - Q_3\,A(\tau)^3 = 0 \quad (52)$$

$$Q_1 = \left[\frac{\alpha^2}{\Pr} + \frac{Ra_{0c}k_c^2}{\alpha^4} + \frac{Le^2 Rask_c^2}{\alpha^4}\right]$$

$$Q_2 = \frac{k_c^2}{\alpha^2}\left[R_2 - 2Ra_{0c}\delta I_1 + VRa_{0c} - RasVLe - \frac{V\alpha^4}{k_c^2}\right]$$

$$Q_3 = \frac{k_c^4}{8\alpha^4}\left[Ra_{0c} - RasLe^3\right] \text{ and}$$

$$I_1(\tau) = \int_{z=0}^{1} f_2\sin^2(\pi z)dz.$$

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It may be difficult to get the analytic solution of the above Ginzburg-Landau equation (52) due to its non-autonomous nature, therefore, it has been solved numerically using the inbuilt function NDSolve of Mathmatica 8.0, subject to the suitable initial condition A(0) = a0, where a0 is the chosen initial amplitude of convection. In our calculations we may assume $R_2 = Ra_{0c}$, to keep the parameters to the minimum.

4 Results and discussion

We consider the effect of temperature modulation on double diffusive convection in a fluid layer that arises when heat and salt make opposing contributions and for $\kappa_T \neq \kappa_S$. Direct mode is preferred in unmodulated case when $\frac{\kappa_S}{\kappa_T} < 1$ and Hopf mode otherwise. We concentrate on the modulated problem for only the direct mode. The focus in the paper is essentially on the effect of modulation on heat and mass transports. In this problem the considered the Ginzburg-Landau equation is nonautonomous. To discuss the results of temperature modulation, we consider the following three types of temperature modulation

- In-phase modulation IPM $\theta = 0$,
- Out-of-phase modulation OPM $\theta = \pi$ and
- Modulation of only the lower boundary MOLB $\theta = -i\infty$.

It is important that a nonlinear study is made if one wants to quantify heat and mass transports which the linear stability theory is unable to do so. External regulation of convection is important in the study of double diffusive convection in a fluid layer. The objective of the paper is to consider such candidates, namely temperature modulation for either enhancing or inhibiting convective heat transport as is required by a real application. The parameters that arise in the problem are Pr, V, Le, Ras, θ, δ, ω and these influence the convective heat and mass transports. The first four parameters relate to the fluid and the structure of the porous medium, and the last three concern the external mechanisms of controlling convection. Positive values of Ras are considered and in such a case, one gets positive values of Ra_T, and these signify the assumption of a situation in which we have cool fresh water overlying warm salty water. Here small amplitude modulation is considered, the value of δ lies around 0.1. Further, the modulation of the boundary temperature assumed to be of low frequency. At low range of frequencies the effect of

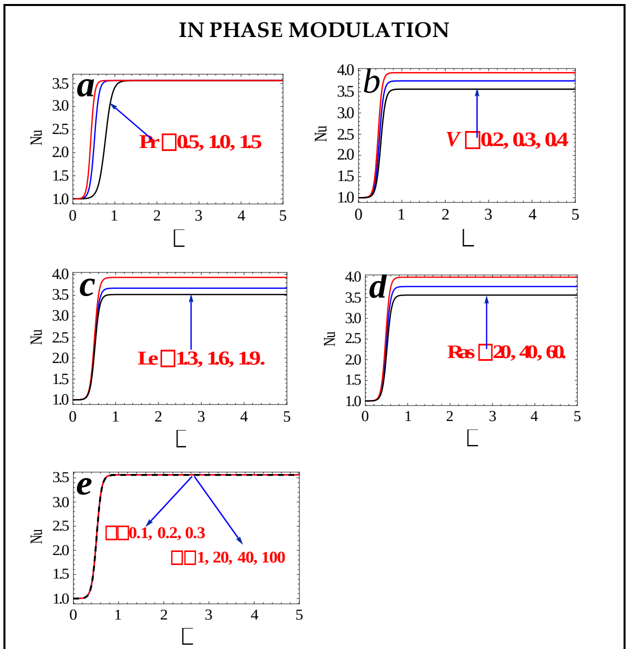

Fig .2:Nu versus τ for (a) Pr (b) V (c) Le (d) Ras (e) δ,ω.

frequency on onset of convection as well as on heat transport is minimal. This assumption is required in order to ensure that the system does not pick up oscillatory convective mode at onset due to modulation in a situation that is conducive otherwise to stationary mode. It is important at this stage to consider the effect of Pr, Le, Ras, δ and ω on the onset of convection.

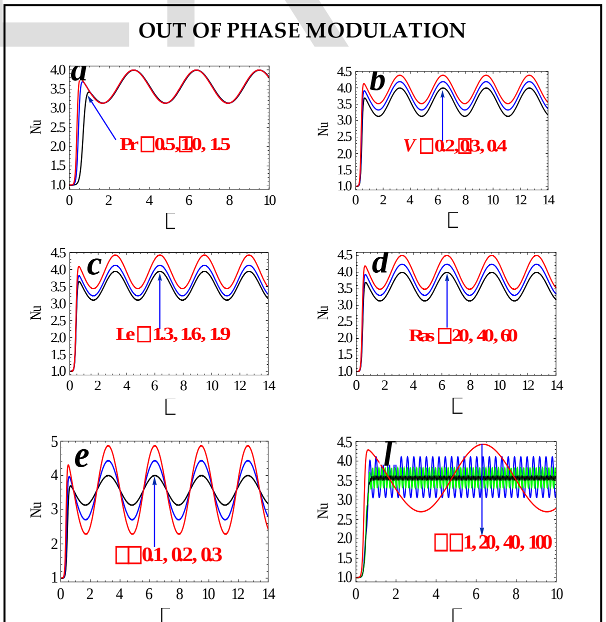

Fig .3:Nu versus τ for (a) Pr (b) V (c) Le (d) Ras (e) δ(f) ω

This has been reported by many investigators earlier who found that:

- $[Ra_T]_{Le=0} < [Ra_T]_{Le\neq0}$

- $[Ra_T]_{Ras=0} < [Ra_T]_{Ras\neq 0}$
- $[Ra_T]_{Pr=0} < [Ra_T]_{Pr\neq 0}$
- and the effect of thermo-rheological parameter V also seen here
- $[Ra_T]_{V=0} < [Ra_T]_{V\neq 0}$

ONLY LOWER BOUNDARY MODULATION

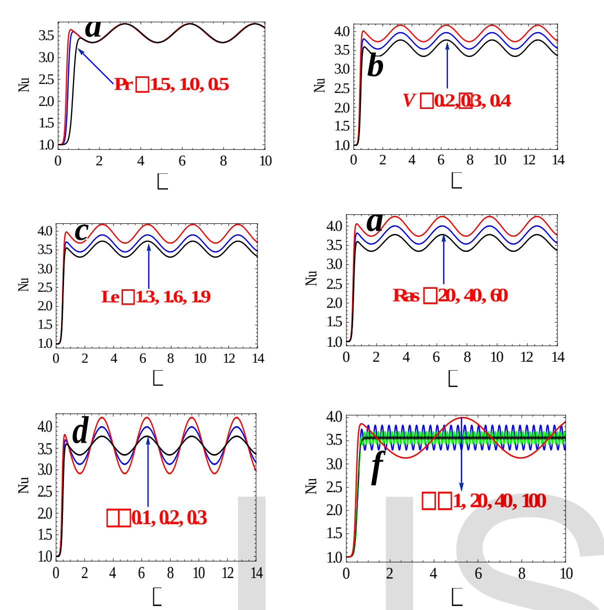

Fig .4:Nu versus τ for (a) Pr (b) V (c) Le (d) Ras (e) δ (f) ω

We fix the values of parameters as Pr=1.0,V=0.2,Le=1.4,Ras=20,δ=0.1 and ω=2.0 and by varying individual parameter, we plot the graphs Nu versus time. The effect of modulation on heat and mass transports is shown in the Figs. (2)-(7). Figs.(2a)-(2d) concerning IPM shows that Nu increases with individual and collective increases in Prandtl number Pr, Temperature dependant viscosity V, Lewis number Le, Solutal Rayleigh number Ras and hence advances the heat transport. But Fig.(2e) shows no effect in the case of amplitude and frequency of modulation. The Nu versus τ curves start with Nu=1, showing the conduction state. As time progresses the value of Nu increases, thus showing that convection is taking place and then finally the curves of Nu level off when time is comparatively large. This result is seen when the amplitude of temperature modulation is quite small. The above pattern in the variation of Pr, V, Le, Ras is also seen in the case of OPM (see Figs.(3a)-(3d)) and LBMO (see Figs.(4a)-(4d)). In the case, OPM and LBMO, however, the Nu versus τ curves are oscillatory. While in the case of OPM Fig.(3e) shows that the effect of δ is to increase the heat transport and the Fig. (3f) shows that the effect of ω is to decrease the heat transport. It is obvious from the figures that:

$[Nu]_{\delta=0.1} < [Nu]_{\delta=0.2} < [Nu]_{\delta=0.3}$

$[Nu]_{\omega=100} < [Nu]_{\omega=40} < [Nu]_{\omega=20} < [Nu]_{\omega=1}$

Further from Fig. (4e)-(4f), we find the similar effect in the case of LBMO.

IN PHASE MODULATION

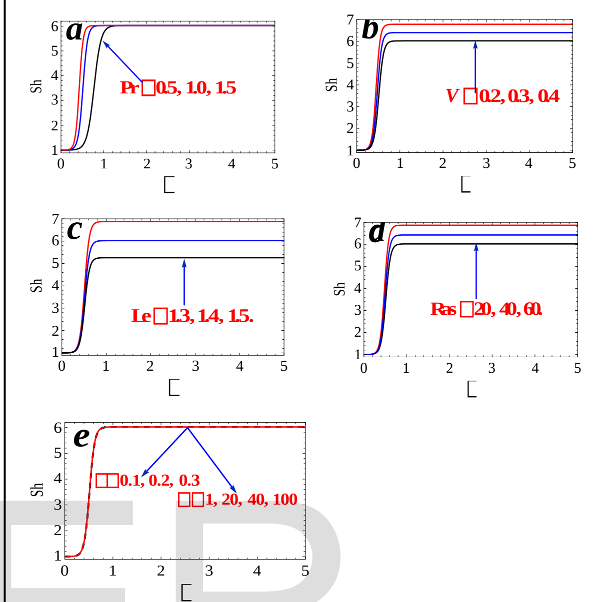

Fig .5:Sh versus τ for (a) Pr (b) V (c) Le (d) Ras (e) δ, ω

From figures (5a)-(5e), (6a)-(6f) and (7a)-(7f) shows the effect of Sherwood number variation with time is similar to Nu variation with time.

OUT OF PHASE MODULATION

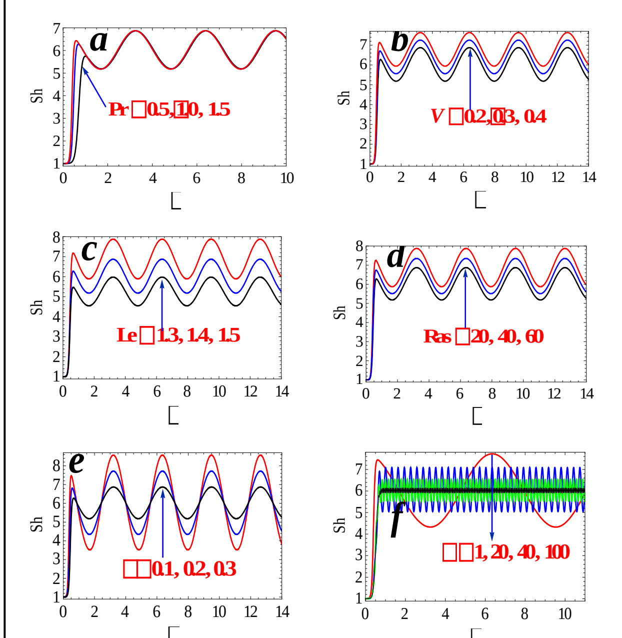

Fig .6:Sh versus τ for (a) Pr (b) V (c) Le (d) Ras (e)δ (f) ω

The general results on Sh, in three types of modulations is similar to those on Nu. The effect of δ Figs. (6e)-(7e) and ω Figs. (6f)-(7f) is same as Nu in the case of OPM and LBMO. It may be noted that in all the above figures only positive values of Ras are considered meaning that only the case of heating from below and salting from below is considered as this favors stationary convection.

ONLY LOWER BOUNDARY MODULATION

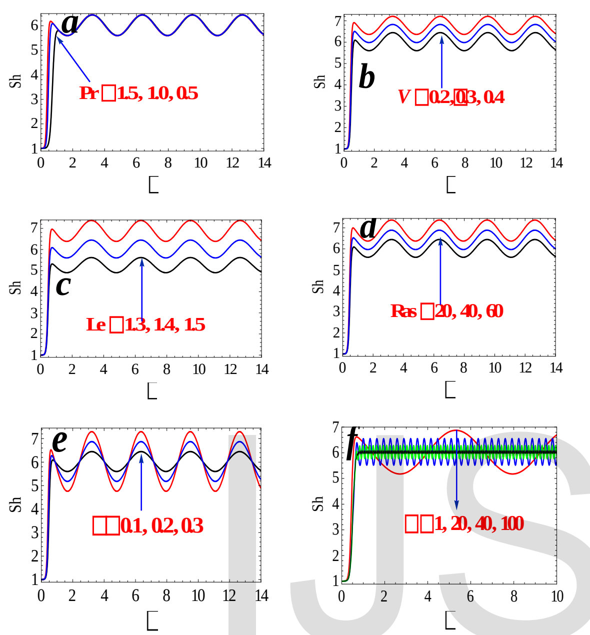

Fig .7:Sh versus τ for (a) Pr (b) V (c) Le (d) Ras (e)δ (f) ω

5 Conclusions

The effect of temperature modulation on weak nonlinear double diffusive convection in a temperature dependant viscous fluid layer is studied using Gingburg-Landau equation. Onset criteria for double diffusive convection for temperature modulation is derived analytically. The following conclusions are drawn

1. Effect of IPM is negligible on heat and mass transport in the system.
2. In the case of IPM, the effect of δ and ω are also found to be negligible on heat and mass transport.
3. In the case of IPM, the values of Nu and Sh increase steadily for intermediate small values of time τ, however become constant when τ is large.
4. Effect of increasing Pr, V, Le, Ras is found to increase Nu and Sh thus increasing heat and mass transfer for all three types of modulations.
5. Effect of increasing δ is to increase the value of Nu and Sh for the case of OPM and LBMO, hence heat and mass transfer.
6. Effect of increasing ω is to decrease the value of Nu and Sh for the case of {OPM} and {LBMO}, hence heat and mass transfer.
7. In the cases of OPM and LBMO, the natures of Nu and Sh remains oscillatory.
8. Initially when τ is small, the values of Nusselt and Sherwood numbers start with 1, corresponding to the conduction state. However τ increases, Nu and Sh also increase, thus increasing the heat and mass transfer.
9. The values of Nu and Sh for LBMO are greater than those in IPM but smaller than those in OPM.

The thermo-rheological model of Nield (1996), gives physically acceptable results, namely, the destabilizing effect of variable viscosity on Rayleigh-B'enard convection and thereby an enhanced heat transport.

The results of this work can be summarized as follows from the Figs. (2-4).

1. $[Nu]_{Pr=0.5} < [Nu]_{Pr=1.0} < [Nu]_{Pr=1.5}$
2. $[Nu]_{V=0.2} < [Nu]_{V=0.3} < [Nu]_{V=0.4}$
3. $[Nu]_{Le=1.3} < [Nu]_{Le=1.6} < [Nu]_{Le=1.9}$
4. $[Nu]_{Ras=20} < [Nu]_{Ras=40} < [Nu]_{Ras=60}$
5. $[Nu]_{\delta=0.1} < [Nu]_{\delta=0.2} < [Nu]_{\delta=0.3}$
6. $[Nu]_{\omega=100} < [Nu]_{\omega=40} < [Nu]_{\omega=20} < [Nu]_{\omega=1}$
   these results are similarly effect for Sh from Figs. (5-7)

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